Investigation of time relativity with respect to Tsiolkovski formula

Tsiolkovski Formula

$$v_2 - v_1 = v_p . Log_e \frac{m_1}{m_2} \qquad \Delta v = v_p . LN . \frac{m_1}{m_2}$$

$$\frac{m_1}{m_2} = e^{\frac{\Delta v}{v_p}} \quad > \quad m_p = m_1 - m_2 \quad > \quad m_p = m_1 - m_1 . e^{-\frac{\Delta v}{v_p}}$$

$$m_p = m_1 . \left[1 - e^{-\frac{\Delta v}{v_p}} \right]$$

$$\frac{m_1}{m_2} = e^{\frac{\Delta v}{v_p}}$$

Why is this rate of change of mass ratio so important

What is the corelation with $\frac{\Delta v_r}{v_p}$

Using an external reference point of an observer on the surface

As the Intertial time reference.

$$\frac{\Delta v}{v_p} = \frac{v_{r2} - v_{r1}}{v_p} = \left[\frac{\frac{L}{T_{Local}}}{\frac{L}{T_{Earth}}} \right] = \left[\frac{L}{T_{Local}} \right] \left[\frac{T_{Earth}}{L} \right] = \left[\frac{T_{Earth}}{T_{Local}} \right]$$

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Then what if at a certain point the time locally at the rocket $[T_{Local}]$ slowed relative to the reference time on Earth $[T_{Earth}]$.

At the Tsiolkovski Point where a specific mass flowrate and rocket velocity creates an energy condition where

$[T_{Local}] = [T_{Earth}]$ and therefore the value of $e^{\frac{T_{Earth}}{T_{Local}}} = 2.7182$.

After this point the value of $e^{\frac{T_{Earth}}{T_{Local}}}$ will increase as $[T_{Local}]$ slows and the relative value of $\frac{\Delta v_r}{v_p}$ increases.

That is a condition exists where the value of $\frac{\Delta v_r}{v_p} = \left[\frac{T_{Earth}}{T_{Local}}\right] \geq 1$

If a similar dimensional analysis of specific impulse is undertaken then an expression for

Impulse Specific Impulse I_{sp} in seconds

$$I_{sp} = \frac{F_{th}}{\dot{m}_p . g} \quad \text{secs}$$

Becomes $\left[\frac{\frac{M}{1} . \frac{L}{T_{Local}^2}}{\frac{M}{T_{Earth}} . \frac{L}{T_{Earth}^2}}\right] = \left[\frac{M}{1}\right]\left[\frac{L}{T_L^2}\right]\left[\frac{T_E}{M}\right]\left[\frac{T_E^2}{L}\right] = \left[\frac{T_E^2}{T_L^2}\right]\left[\frac{T_E}{1}\right]$

Then a reduction in $[T_{Local}]$ would also alter I_{sp}

For this condition to exist the rate of change of mass of the projectile has to create its own space time horizon.

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