Derivation of Tsiolkovsli's Rocket Formula.

Imagine a rocket of mass 'M' and velocity 'u' relative to a stationary observer. Then let 'v' be the exhaust velocity of the rocket jet as it leaves the nozzle.
Now if a small amount of fuel burns in the combustion chamber of the rocket motor. It produces a high energy gas stream which expands and leaves the the rocket nozzle to enter the atmosphere. We can say that the rocket mass has been effectively reduced by a small amount dM whilst the exhaust jet has been increased by the same amount dM .

We can then create an expression for the momentum of the rocket and the exhaust jet after a small increment in time dt :

$$\Delta_{\text{Rocket}} = (M - dM)du \qquad\qquad 1$$

$$\Delta_{\text{Exhaust}} = dM(u + v) \qquad\qquad 2$$

Applying Newtons law for the 'Conservation of momentum' to equations 1 and 2 we have.

$$dM(u + v) + du(M - dM) = 0 \qquad\qquad 3$$

Initially the gas stream jet velocity will be far greater than that of the rocket .Therefore

$u <<< v$ So we say $u => 0$ tends to zero or is insignificant relative to the size of velocity v
and if the elemental mass of exhaust gas lost relative to that of the whole rocket very small.
Then we can say

$M >>> dM$ or $dM => 0.$

Re writing equation 3 we get

$$dM.v + du.M = 0 \qquad\qquad 4$$

Rearranging Eq4 for du

$$du = \frac{-v.dM}{M} \qquad\qquad 5$$

Integrating between definite limits from the Initial point 1 to point 2.after an elemental time dt when du and dM occur. We get

$$\int_{U1}^{U2} 1.du = -v \int_{M1}^{M2} \frac{dM'}{M'} \qquad\qquad 6$$

$$[u_1 - u_2] = -v [\text{Ln } M_1 - \text{Ln } M_2] = -v.\text{Ln}\frac{M1}{M2}$$

Multiply both sides of the equation by –1 and rearranging for u_2

$$u_2 = v.\text{Ln}\frac{M1}{M2} + u_1$$