Propulsion Force.

Imagine a rocket of mass m_1 being propelled at a velocity v by a reaction jet of hot exhaust gas of mass m_{exh} and velocity v_{exh} Then applying *Newton's law* for the conservation of momentum we get :

Momentum

$$m_1.\vec{v} = m_{exh}.\vec{v}_{exh} \tag{1}$$

Force

Thrust force can be defined as the rate at which impulse I changes with time

$$\vec{F} = \frac{d\vec{I}}{dt} \tag{2}$$

$$\vec{F} = \frac{d}{dt}.m_p.\vec{v}_p$$

$$\vec{F} = \dot{m}_p.\vec{v}_p \tag{3}$$

Where $\dot{m}_p$ is the propellant or fuel mass flowrate and v_p its exhaust velocity

Impulse

Specific Impulse Isp in seconds

$$I_{sp} = \frac{F_{th}}{\dot{m}_p.g} \tag{4}$$

Rewriting the momentum equation 1

$$m_1.\frac{dv}{dt} = \frac{dm_p}{dt}.v_p \tag{5}$$

Because the rate of change of the rocket mass dm

Is equal to the loss in mass of the fuel or propellant dm_p then $dm_1 = dm_p = dm_{exh}$

$dm_1 = dm_p$

Substituting this into Eq 5 we get

$$m_1 . \frac{dv}{dt} = \frac{dm_p}{dt} . v_p \qquad 5$$

$$dv = -v_p . \frac{dm_1}{m_1} \qquad 6$$

Integrating both sides over a fixed event time when initial rocket mass m_1 has reduced to m.

$$\int_1^2 dv = -v_p . \int_1^2 \frac{1}{m_1} . dm_1$$

$$v_2 - v_1 = -v_p . [Log_e m_2 - Log_e m_1]$$

Tsiolkovski Formula

$$v_2 - v_1 = v_p . Log_e \frac{m_1}{m_2} \qquad 7$$

$$\Delta v = v_p . LN . \frac{m_1}{m_2}$$

Using the relationship that if $y = Log_n x$ $\qquad x = e^y$

Then $\Delta v = v_p . LN . \frac{m_1}{m_2}$

Can be expressed in the form $y = Log_n x$

$$\frac{\Delta v}{v_p} = LN \frac{m_1}{m_2}$$

So it follows that

$\frac{m_1}{m_2} = e^{\frac{\Delta v}{v_p}}$ or $m_2 = m_1 . e^{-\frac{\Delta v}{v_p}}$

Initially the mass relationship is given by

$$m_p = m_1 - m_2$$

Then

$$m_p = m_1 - m_1 . e^{-\frac{\Delta v}{v_p}}$$

$$m_p = m_1 . \left[1 - e^{-\frac{\Delta v}{v_p}} \right] \qquad 8$$

