

Propulsion Force.

Imagine a rocket of mass m_I being propelled at a velocity v by a reaction jet of hot exhaust gas of mass m_{exh} and velocity $v_{ex.h}$. Then applying *Newton's law* for the conservation of momentum we get :

Momentum

$$m_1 \cdot \vec{v} = m_{exh} \cdot \vec{v}_{exh} \quad 1$$

Force

Thrust force can be defined as the rate at which impulse I changes with time

$$\vec{F} = \frac{d\vec{I}}{dt} \quad 2$$

$$\vec{F} = \frac{d}{dt} \cdot m_p \cdot \vec{v}_p$$

$$\vec{F} = \dot{m}_p \cdot \vec{v}_p \quad 3$$

Where \dot{m}_p is the propellant or fuel mass flowrate and v_p its exhaust velocity

Impulse

Specific Impulse I_{sp} in seconds

$$I_{sp} = \frac{F_{th}}{\dot{m}_p \cdot g} \quad 4$$

Rewriting the momentum equation 1

$$m_1 \cdot \frac{dv}{dt} = \frac{dm_p}{dt} \cdot v_p \quad 5$$

Because the rate of change of the rocket mass dm_1

Is equal to the loss in mass of the fuel or propellant dm_p then $dm_1 = dm_p = dm_{exh}$

$$dm_1 = dm_p$$

Sustituting this into Eq 5 we get

$$m_1 \cdot \frac{dv}{dt} = \frac{dm_p}{dt} \cdot v_p \quad 5$$

$$dv = -v_p \cdot \frac{dm_1}{m_1} \quad 6$$

Integrating both sides over a fixed event time when initial rocket mass m_1 has reduced to m .

$$\int_1^2 dv = -v_p \cdot \int_1^2 \frac{1}{m_1} \cdot dm_1$$

$$v_2 - v_1 = -v_p \cdot [\text{Log}_e m_2 - \text{Log}_e m_1]$$

Tsiolkovski Formula

$$v_2 - v_1 = v_p \cdot \text{Log}_e \frac{m_1}{m_2} \quad 7$$

$$\Delta v = v_p \cdot \text{LN} \cdot \frac{m_1}{m_2}$$

Using the relationship that if $y = \text{Log}_n x$

$$x = e^y$$

Then
$$\Delta v = v_p \cdot LN \cdot \frac{m_1}{m_2}$$

Can be expressed in the form $y = \text{Log}_n x$

$$\frac{\Delta v}{v_p} = LN \frac{m_1}{m_2}$$

So it follows that

$$\frac{m_1}{m_2} = e^{\frac{\Delta v}{v_p}} \quad \text{or} \quad m_2 = m_1 \cdot e^{-\frac{\Delta v}{v_p}}$$

Initially the mass relationship is given by

$$m_p = m_1 - m_2$$

Then

$$m_p = m_1 - m_1 \cdot e^{-\frac{\Delta v}{v_p}}$$

$$m_p = m_1 \cdot \left[1 - e^{-\frac{\Delta v}{v_p}} \right]$$